

## The Effectiveness of Manipulatives During 1:1 Mathematics Intervention

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## The Effectiveness of Manipulatives During 1:1 Mathematics Intervention

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### Abstract

The use of manipulatives in elementary schools is one beneficial method that has been utilized to help students grasp abstract mathematical concepts. To examine the effectiveness of implementing mathematical manipulatives, researchers designed a seven-week intervention for ten low-achieving third- and fourth-graders. Topics focused on the National Council of Teachers of Mathematics's standard related to number and operations. Data was collected based on observation with the use of a Likert scale measuring each student's achievement with the chosen and designed manipulatives. Success was defined as students being able to complete assessments using the manipulatives without aid and with little to no computation errors. Results suggested that there was a 70% success rate in third-grade students and an 84% success rate for fourth-graders. It was concluded that the manipulatives helped students complete number sense operations when used in tutoring but may not have aided in students' abstract understanding of mathematics concepts.

Keywords: manipulatives, elementary, intervention, mathematics

## Introduction

This research aims to understand the effectiveness of utilizing mathematical manipulatives in a one-on-one environment for upper-elementary students that may benefit from intervention. While there is a significant amount of research on the effectiveness of mathematics manipulatives and tutoring in elementary subjects, there is less research available on their combined impact. Additionally, research on the effectiveness of manipulatives for lower-elementary students is more readily available than for upper-elementary grades. Through formal observation, this research will evaluate the effectiveness of tutoring with mathematical manipulatives for upper-elementary students who have been identified as below grade-level in mathematics. For the purposes of the research, a manipulative is a tool that students can hold and move in order to help create a connection and understanding of an abstract concept. The tutoring sessions were designed to accommodate students, teachers, and researchers, lasting twenty minutes per student, once a week, for a total of seven weeks. Mathematical concepts with the use of manipulatives explored in this study include addition, subtraction, multiplication, and division. The key research question is: What is the impact of mathematical manipulatives in a one-on-one intervention for lower-achieving third- and fourth-grade students?

## Literature Review Definition of Manipulatives

Due to the breadth of concepts in mathematics—such as number and operations, algebra, geometry, measurement, and data analysis and probability (NCTM, 2000)—manipulatives can be used differently depending on the concept. Manipulatives are often used by mathematics teachers in elementary grades to give students a concrete representation of a mathematical concept. Across the literature, however, there are generally-accepted definitions of what qualifies as a manipulative. McNeil and Jarvin (2007) define manipulatives as “concrete objects used to help students understand abstract concepts, such as those often encountered in the domain of mathematics” (p. 310). Similarly, Moore (2014) defines manipulatives as “physical objects that students and teachers can use to illustrate and discover mathematical concepts, whether made specifically for mathematics (e.g., connecting cubes) or for other purposes (e.g., buttons)” (p. 24). Whether a teacher buys a product specifically made to teach mathematics, or if they make use of an ordinary item and develop it to teach mathematics, there is significant evidence indicating that manipulatives can improve understanding of mathematical concepts.

## Discussion of manipulatives

Both researchers and developmental theorists agree that the physical use of manipulatives leads to a deeper understanding of abstract concepts; therefore, students retain the material more successfully. Developmental theorists (Bruner, 1964; Montessori, 1964; Piaget, 1962) argue that young children are believed to obtain cognitive benefits from exploring mathematical concepts with manipulatives because they are assumed to have greater dependency on physically interacting with their environment to construct meaning (Carbonneau, Marley, & Selig, 2013). Therefore, elementary students reside in the developmental stages in which mathematical manipulatives aid in constructing their meaning of mathematics and the physical action of manipulatives can enhance this understanding. McNeil and Jarvin (2007) reinforce the same idea in research on how manipulatives involve physical actions, which is important because physical action “has been shown to enhance memory and understanding” (p. 311) in mathematics education. This concept can be important when considering kinesthetic learners in the classroom and aiding mathematical understanding.

In educational research, the implementation of manipulatives in mathematics instruction have been found to improve students’ performances. Moyer (2001) draws on research from the past 40 years to demonstrate that “students who use manipulatives during mathematics instruction outperform students who do not” (p. 177). It is impressive that such a large consensus exists among educational researchers. A study completed by Fuson and Briars (1990) found that first- and second-graders who used manipulatives, such as base-ten blocks, “showed multidigit addition and subtraction computation performance that was very considerably above that shown by third-graders receiving traditional instruction” (p. 195). The substantial difference between performance that can be achieved through the use of manipulatives illustrates the positive effects that manipulatives can have for students. Research shows that students who receive mathematics instruction with manipulatives often outperform those who receive traditional instruction without them (Moyer, 2001 & Fuson & Briars, 1990).

While manipulatives aid directly in the cognitive processes, they also increase students’ interest and enjoyment of mathematics (Morre, 2014). This can be beneficial for students who are discouraged by traditional teaching techniques and who lack an engagement with mathematics instruction. Thus, manipulatives enable teachers to reach students who may face difficulties with typical instruction, such as low achievers, English Language Learners,

and students with learning disabilities (Hand2mind, n.d.). The increased engagement and interest that manipulatives provide make it helpful for those students who too often are discouraged from mathematics due to their inability to grasp abstract concepts at an early age. Manipulatives can also serve as a communication point for students, especially those with lower achievement. Ojose and Sutton (2009) state that “manipulatives not only allow students to construct their own cognitive models for abstract mathematical ideas and processes, it also provides a common language with which to communicate these models to the teacher and other students” (p. 4). This can encourage a sense of community in a mathematics classroom and provide a voice and communication platform for students who historically struggle with abstract mathematical concepts.

As the impact of manipulatives are investigated, it is important to acknowledge the problems that can come with their implementation in the classroom. Moyer (2001) acknowledges that sometimes, students learn how to use manipulatives in a rote manner with little or no learning of the mathematical concept behind the procedure. Similarly, McNeil and Jarvin (2007) point out that even when students are able to use manipulatives to demonstrate an understanding of mathematical concepts, they may still fail to apply that knowledge when solving problems unless they are explicitly reminded to think about the manipulatives. Therefore, a common drawback to implementing manipulatives is that—although students may be able to complete tasks they could not previously without the manipulative—they still do not have a conceptual understanding of what they are doing or why. Moyer (2001) also states that this then means that students are unable to link the manipulatives to abstract symbols

### **Implementation of manipulatives**

When deciding to use manipulatives to assist students in grasping mathematical concepts, it must be recognized that students do not discover or understand these concepts simply by manipulating concrete materials. As McNeil and Jarvin (2007) point out, “a given manipulative needs to be represented not only as an object in its own right, but also as a symbol of a mathematical concept or procedure” (p. 313). It is important to remember that while introducing these objects does not guarantee understanding, it does provide an environment for discussion, communication, and reflection (Picciotto, 1998). Teachers must intervene frequently as part of the instructional process to help students recognize the underlying mathematical ideas as well as create connections between the students’ manipulation and the corresponding mathematical symbols or processes

(Sutton & Krueger, 2002). Teachers must be sure to consider the processes and aspects of manipulatives, as well as consider their role as the facilitator, when deciding how to use them effectively in the classroom.

One aspect to consider is the amount of time students spend interacting with the tools. Moore (2014) states that it is essential that manipulatives be a frequent element of mathematics practice for them to be effective. Also, an article by Moore (2014) found that “when students are exposed to hands-on learning on a weekly rather than a monthly basis, they prove to be 72% of a grade level ahead in mathematics” (p. 27). More research is needed on the topic of how time affects the success of manipulatives in upper elementary grades. However, time can be an important factor in the effectiveness of manipulatives and has thus far proved to need regular implementation over a long period of time in order to impact student comprehension.

### **Method Selection of Participants**

Due to an already established relationship between a teacher education program in a mid-Atlantic state and a local rural Title 1 elementary school, student participants were purposefully-selected third- and fourth-graders. It was in the interest of the researchers to support the highest-need students in the school; therefore, the classroom teachers and principal selected students who would benefit the most from mathematics instruction, as they were below grade-level. Permissions were obtained by the district superintendent, principal, classroom teachers, and parents. One researcher worked with the five third-grade participants, while the other researcher worked with the five fourth-grade participants.

### **Weekly instruction**

Researchers met individually with students once a week for twenty minutes. These times for tutoring were provided to researchers so that the intervention did not impact in-class instructional time. The seven weeks of intervention were instructional and introduced the student to a manipulative that was either handmade or purchased by the researcher. Researchers made weekly lesson plans designed to focus on students’ most pressing needs, which were specified by the students’ teachers. The format of the lessons were as follows: introduce the manipulative, explain how they would use it and how it related to the concept they were learning, demonstrate/model example problems using the manipulative, perform guided practice with the student using manipulative, then perform independent practice, and a short assessment at the end.

All of the topics focused on the NCTM's number and operations strand.

### 3rd Grade:

	Subject Focus	Manipulative
Week 1	Addition and subtraction	Fact Family Triangles
Week 2	Addition	Ladder
Week 3	Place value and addition	Paper Clips
Week 4	Addition and subtraction	Number Sentences with Jenga Blocks
Week 5	Addition with double-digit answers	Number Sense with Counters
Week 6	Multiplication	Legos
Week 7	Subtraction	Straws

### 4th Grade:

	Subject Focus	Manipulative
Week 1	Multiplication and division	Factor Triangles
Week 2	Multiplication - Arrays	100 chart and arrays
Week 3	Multiplication - Repeated addition	Legos
Week 4	Multiplication - Repeated addition	Pipe cleaners with beads
Week 5	Multiplication	Popsicle stick ladder
Week 6	Long division	Steps Slider
Week 7	Division - Equal groups	Play-doh

### Measuring student success

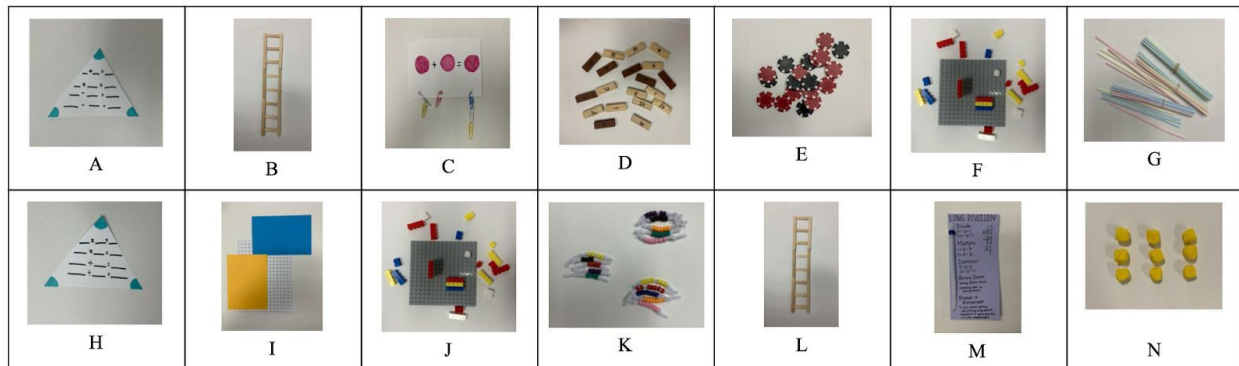
Researchers observed students and used a rubric to measure how successful the students were with the manipulatives. Students did not see this rubric. Researchers evaluated students immediately after their assessment using the rubric. Each third-grade student received the same assessment questions each week pertaining to their content, and fourth-grade students received a separate set of assessment questions based on their content. Participants were given a score on the rubric immediately after completing the assessment. The rubric is as follows:

The student completed the task correctly and independently. (4)	The student completed the task independently, but with errors. (3)	The student needed some guidance in order to complete the task. (2)	The student did not understand the task and was unable to complete it. (1)
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### Manipulatives

In total, there were fourteen manipulatives used: seven for third-grade students and seven for fourth-grade students. Some manipulatives were used by both sets of students and modified to fit the needs of the grade being worked with. The following chart comprises all of the manipulatives used in this research. The descriptions of each manipulative are broken into categories based on the grade level the manipulatives were used for and correspond with the letter attached to the manipulatives in the chart.

## Manipulatives



### Third-grade manipulatives

Manipulative A, the fact family triangles, included laminated triangles that students could write on to visualize the relationships between three numbers. Students wrote the three numbers in separate corners of the triangle. In the middle of the triangle, they wrote different equations involving the three numbers. For example, if a student was given the numbers 1, 2, and 3, the equations would be  $1+2=3$ ,  $2+1=3$ ,  $3-1=2$ , and  $3-2=1$ .

Manipulative B, the ladder manipulative, was used for basic addition facts to help develop number sense. The ladder was created out of popsicle sticks. When given an addition problem, students could put the ladder on a piece of paper and work out the problem by going up the ladder. For example, if a student was given  $5+4$ , they would put their ladder down on the paper. At the bottom “rung,” students would write the 5, then they would write, 6, 7, 8, and 9 in the following “rungs.” Doing so allowed students to visualize the addition problem, rather than counting on their fingers.

Manipulative C, the paper clip manipulative, was used to practice addition. Paper clips were grouped into chains of ten, while some were left on their own. Students also used laminated notecards with two holes punched in the bottom so the paper clips could be attached. When given an addition problem, students would write the equation on the notecard and attach the number of paper clips. If the problem was  $10+5$ , students would put 10 paper clips beneath where they wrote 10, and 5 paper clips beneath the 5. Students could then count the paper clips to solve the problem and visually see the solution.

Manipulative D, Jenga blocks, was used to form number sentences. The Jenga blocks had the numbers 1-25 written on them, as well as  $+$ ,  $-$ , and  $=$ . Students would pull

whatever numbers they wanted and create a sentence with them. For example, they might have chosen 12, 8,  $-$ , 4, and  $=$  to create  $12-4=8$ . The task was open-ended and allowed students to make connections between numbers on their own. For assessment, students were asked to show three different number sentences that had the same answer.

Manipulative E allowed students to continue working on number sense and addition using circular counters. The students had two sets of different-colored circles and were shown that when given an addition problem, the first number would be one color and the second number would be the other color. If they had white and green counters and were given the problem  $1+2$ , they would get one white counter and two green counters and then count all three to get the answer. Students could see that different equations can still lead to the same answer by observing the different distributions of colors while still having the same total. Students were asked to find different equations that had the same answer ( $6+6$ ,  $5+7$ ,  $8+4$ , etc.).

Manipulative F was used for multiplication. After week 5, many students requested help on multiplication, so students were given a tub of Lego pieces and a flat Lego board on which to build. Using the Lego pieces, students could build sideways to form an array for multiplication factors, build up to see how multiplication stacks up on top of the last factor, or form equal groups.

Manipulative G consisted of straws rubber-banded together in groups of ten. Students were also given some individual straws. Week 7 focused on subtraction, also by student request. When given a subtraction problem, students would gather the number of straws needed and then take away the given number, regrouping as needed to solve the problem.

### Fourth-grade manipulatives

Manipulative H had a similar design to Manipulative A. During week one, the fact family triangles were used with multiplication and division. For example, if students were given the numbers 2, 3, 6, then they would write  $2 \times 3 = 6$ ,  $3 \times 2 = 6$ ,  $6 \div 2 = 3$ ,  $6 \div 3 = 2$ .

Manipulative I was an array strategy presented to students to help them conceptualize multiplication. To do this, a centimeter one-hundreds chart was laminated, and two note cards were given to the students. Students would create an array by placing the notecards in a rectangular shape to represent a multiplication problem. For example, if given the problem  $3 \times 4$ , then students would count over 3 squares and down 4 and put the note-cards on the chart to make a rectangle at those spots. For assessment, this week focused on factors of 6 and 7.

Manipulative J was the same Lego manipulative used with third grade to help visualize multiplication as repeated addition. Students were given a base and multiple Legos of different lengths. Students were instructed to stack Legos on top of one another. This week focused on the multiplication facts for multiples of 2, 3, 4, and 5. To choose the length of the Lego, students were told to use the first number in the expression and then use the second number to decide how many to stack up. For example, if the problem was  $5 \times 3$ , they would use a Lego 5 dots long, and put 3 together.

Manipulative K was pipe cleaners with beads. Repeated addition had thus far stuck best with students and was therefore chosen to be represented in another manner in order to work on factors that had proven difficult. To do this, pipe cleaners were strung with beads in sets of 4, 6, and 7. Students would be given a problem, such as  $4 \times 3$ , and would line up 3 pipe cleaners that had 4 beads on them to represent the strategy of repeated addition.

Manipulative L was a popsicle stick ladder created to demonstrate consecutive multiplication factors, similar to the ladder used with third grade. Students would write the multiplication expression beside the ladder, such as  $3 \times 2$ , and the answer inside the rungs, such as 6. This would go at the bottom and students would work their way up, such as  $3 \times 3$ ,  $3 \times 4$ ,  $3 \times 5$ , etc., and move the ladder up to keep going. The focus for this week was factors of 6, 7, 8 and 9.

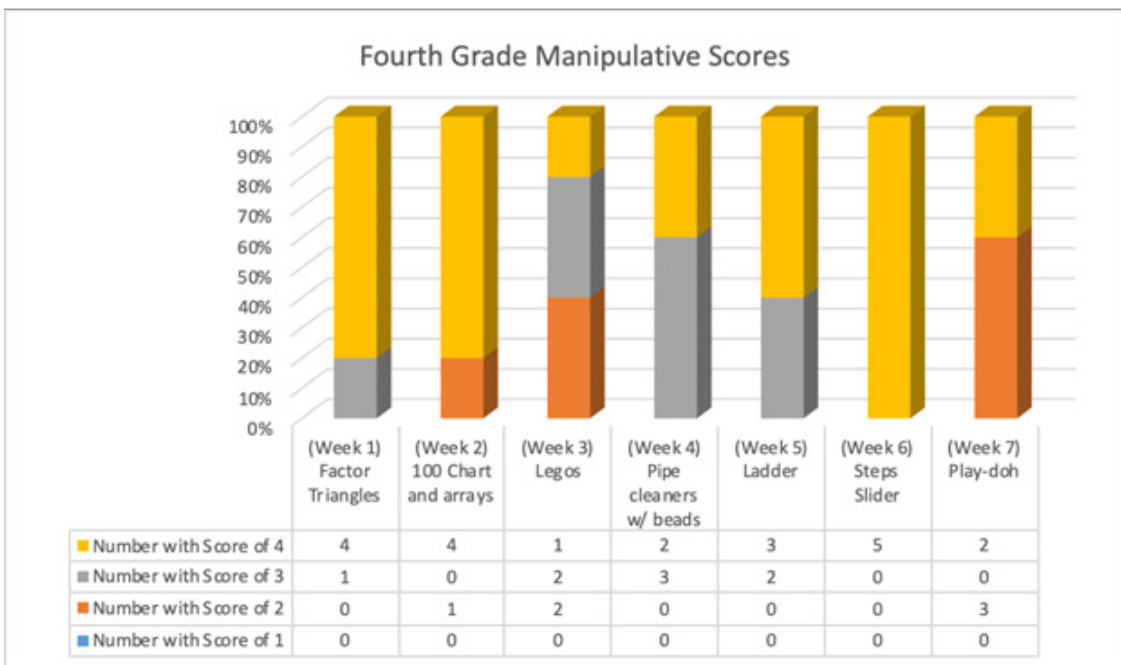
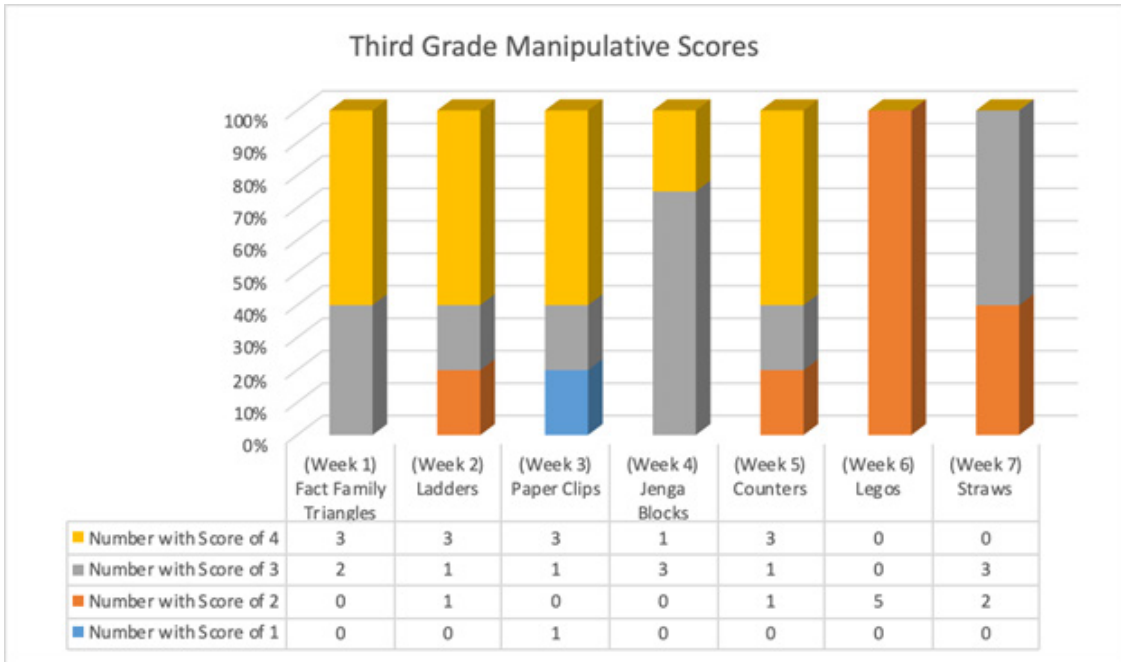
Manipulative M was a step slider that shifted the focus from multiplication to division. Students were given a long-division slider that they could move a bead down as they went through the steps of a long-division problem. This was created with a bookmark-size paper with instructions, an example problem on the side, and two

hole-punches with a pipe cleaner through them containing a bead to slide beside the steps. Students worked through a long-division problem by sliding the bead to keep track of what step they were on and what came next.

Manipulative N was Play-doh, used to represent the strategy of equal groups. Students would separate the Play-doh into small balls and then put them in groups to represent the division problem. For example, if given  $24 \div 4$ , students would make 4 groups and add Play-doh balls one at a time to each group until they got to 24. They would then count how many balls were in each group and that would be the solution. This focused on the divisors of 3, 4, 5, and 6.

### Results

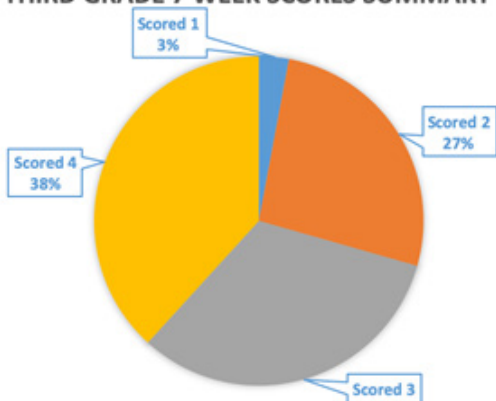
Data was analyzed based on observation of students' ability to use the manipulative given each week. The bar graph titled "Third Grade Manipulative Scores" presents the results of each manipulative and rubric score that each third-grade student received, and the bar graph titled "Fourth Grade Manipulative Scores" presents these results for fourth-grade students. In comparison, the pie charts titled "Third Grade 7 Week Scores Summary" and "Fourth Grade 7 Week Scores Summary" present the scores for all seven weeks, comparing the overall percentage of each rubric score received.



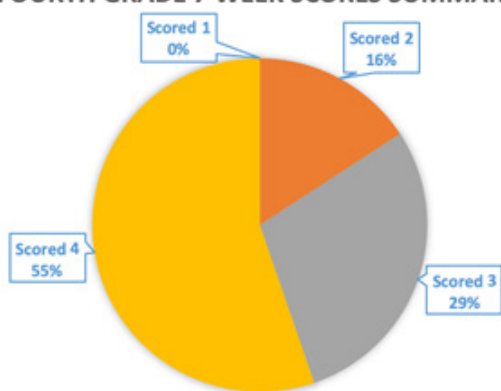


Findings were compared by completion without aid (scores of 3 and 4) and completion with aid (scores of 1 and 2). Therefore, the data illustrates that for both third and fourth grade, the following manipulatives were 100% successful without the aid of researchers: fact family triangles, jenga blocks, factor triangles, 100 chart and arrays, pipe cleaners with beads, ladder, and steps slider. For third-grade students, the ladder paper clips and counters manipulatives had an 80% success rate without aid and 20% completion rate with aid. The Lego and straws manipulatives for third-grade students had 0% success without aid and therefore a 100% completion rate with aid. For fourth-grade students, Legos had a 60% success rate without aid and a 40% success rate with aid. The Play-doh manipulative for fourth-graders resulted in a 40% success rate without aid and a 60% completion rate with aid.

THIRD GRADE 7 WEEK SCORES SUMMARY



FOURTH GRADE 7 WEEK SCORES SUMMARY



Overall, the most common rubric score observed in both grades was a four—38% in third grade and 55% in fourth grade. The second most common rubric score was a three, which was scored a total of 32% of the time in third-grade students and 29% in fourth-grade ones. In total, there was a 70% success rate without aid from researchers in third-graders and 84% in fourth-graders. Twenty-seven percent of third-graders and 16% of fourth-graders achieved a score of 2. In third grade, a rubric score of 1 occurred 3% throughout the course of the intervention. Therefore, there was a 30% completion total of manipulative tasks with aid in third-grade students and 16% in fourth-grade students.

### Discussion

The purpose of this intervention study was to analyze and observe the effectiveness of utilizing mathematics manipulatives in a one-on-one environment for upper-elementary students who may benefit from intervention. The results present a greater success rate without the aid of researchers when using manipulatives. Between both the third- and fourth-grade students, researchers concluded that students were able to mimic the modeling of the manipulative; however, their conceptual understanding of the content was unclear. This observation supports Moyer's (2001) research, which states that at times students are unable to link their actions with manipulatives to the corresponding abstract symbols. It was difficult to observe whether students had a deep understanding of the content and how the manipulative tied into the content or if they just methodically followed the researcher's instructions. For example, the third-graders who received a score of four during the fact family triangle week seemed to know where to place the numbers without actually understanding the relationship between them. This was also obvious with the fourth-grade students, as they were able to fill in the chart, but if asked the answer to the multiplication fact without looking at what they had just written, they were unable to produce the number. The research of McNeil and Jarvin (2007) does support the result that, although students may be able to demonstrate an understanding of the mathematical concept, they fail to apply this without the manipulative to use. While there can be value in seeing the patterns in manipulatives, researchers hoped students would develop a stronger number sense and be able to apply their knowledge even if they did not have the manipulative with them.

The students selected for the study were classified as below grade-level by their teachers. Researchers were given little background information on the students, other than the fact that they had poor number sense and generally

struggled in mathematics. Due to the lack of number sense the selected students exhibited, researchers had to try to build the foundation, and it became difficult to determine if the manipulative was not beneficial or if the student needed a better foundation in number sense for the manipulative to work.

Another observation researchers made was a lack of connection between number sense topics that are related to one another, such as addition and multiplication or multiplication and division. When manipulatives for division were introduced to the fourth-grade students, the aspect that was most difficult for them was multiplying in long division. For example, if students had to divide 21 by 7 and find the number to write about the long-division symbol, they could not understand that it was three because  $7 \times 3 = 21$ . Researchers do believe, however, that students did start to build a toolbox of strategies to solve number sense expressions when allowed to use manipulatives that they worked with best. The third-grade students, for example, began making connections between different manipulatives. The students noticed that the Jenga blocks manipulative was similar to the circular counters, as both showed that different numbers could lead to the same sum. The fourth-grade students also began using the manipulatives they most connected with in class and showed classmates how they could use the manipulatives too. Students also demonstrated connections between manipulatives and using past manipulatives to help them understand and solve the new manipulatives they were given.

Researchers also noted that students tended to do well with all of the manipulatives when given numbers with which they were familiar. For example, the fourth graders knew some portions of the times tables better than others and used their prior knowledge when working with the manipulative. Most fourth-graders knew their factors of 2, 5, and 9, either through memorization or other strategies they had been taught. The manipulatives seemed to do very little to help when students were working with a more well-known pattern, but proved to be a beneficial tool when working with the less familiar numbers. Researchers, however, tried to use these familiar digits to familiarize students with the manipulatives and then build in more challenging digits as the intervention continued, hoping students had developed some number sense from previous manipulatives. The third-grade students were often familiar with patterns of 5s and 10s when using addition and subtraction. When given problems with those patterns, their success was far higher than when given a variety of numbers or patterns they were less familiar with, like 7s or 9s for third-grade students and factors of 6, 7, and 8 for fourth-grade students.

Researchers noticed that students, specifically the third-grade group, started displaying a sense of agency as they grew more comfortable. The main focus for the third-grade students was growing number sense through addition, with the first five weeks being solely focused on that. After the fifth week, however, multiple students expressed that they needed help with multiplication and requested to spend the next week practicing multiplication. While students still could have benefited from additional practice with addition, the researchers chose to acknowledge the student request and practice multiplication, using repeated addition as a potential strategy for students. Therefore, students still had the opportunity to improve their addition skills. The sense of agency that these students developed was interesting, and it may be worth considering what caused the development of agency in future research.

### Limitations

The circumstances of the study only allowed researchers to have twenty minutes with students over eight weeks, and only seven of those weeks included instructional time. Time restraints were partially due to difficulty matching researcher schedules with participants' school schedules. Students were not able to miss certain parts of their day, such as PE, music, art, library, lunch, and certain instructional periods. Teachers did not feel comfortable having students miss more than twenty minutes of instructional time in the classroom. By the end of the study, students only had a little more than two hours of one-on-one instruction with researchers. Previous research recommends that manipulatives should be used over long periods of time in order to have any positive effect on student success. Two hours was simply not enough time to make a large impact on any participants.

Researchers had no prior knowledge of the students chosen for intervention and their needs, other than the fact that they were considered below grade level. Therefore, manipulatives were chosen on general topics that the teachers had designated as needing work and would benefit students' overall mathematics abilities. This became a challenge in creating the first manipulatives and deciding what specific areas students needed help in. As such, researchers chose strategies all students could use. While this was the best strategy determined at the beginning of the research, researchers found that it would have been beneficial to know more about the students—how they had previously worked with mathematics concepts and their levels of understanding. Researchers did not know enough about each student to design manipulatives specifically tailored to their needs.

### Further research

After conducting this research, some areas that need attention and research in order to achieve better understanding have surfaced. A longer study should be done on the impact of manipulatives over an extended period of time. This could be done with a focus on longer sessions of one-on-one intervention or the implementation of manipulatives in classroom instruction. Research should also be done on connecting types of manipulatives from concrete understanding to abstract concepts. As a result of this study, research should be done on the relationship between tutoring programs and teachers and the implementation of strategies from the tutoring into the classroom. Researchers also wondered how well the students remembered the manipulatives and if they fostered long-term understanding of the topics. In further research, a longer study should evaluate the effectiveness of manipulative use on long-term understanding.

### Conclusion

The research conducted was designed to understand the effectiveness of implementing mathematical manipulatives in a one-on-one environment for third- and fourth-grade students. Results from this observational research suggest that manipulatives can benefit students and provide them with a tool to solve number and operation mathematical concepts. Researchers recognized that participating students could perform the required tasks with the manipulatives, but there appeared to be a disconnect between the concrete manipulatives and the abstract concepts they were designed for. This has been a noted concern with manipulatives, as shown in research by Moyer (2001). While manipulatives can provide students with a tool to solve problems, it is unclear from this research whether there are long-term benefits on overall number sense. Researchers also observed that students were able to solve mathematical problems with the manipulatives in front of them, but could not solve the same problems without the use of manipulatives in later sessions. They also had difficulty drawing connections between the manipulatives used from week to week. This observation is supported by the research of McNeil and Jarvin (2007), who also found that students may be able to use the manipulative correctly, but not be able to apply the same concept without the manipulative. What is certain, though, is that manipulatives provide lower-achieving students in upper-elementary grades with hands-on tools to aid them in developing comprehension of number sense and operations by providing them with a variety of strategies.

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